EXERCISES 3, QUESTION 2

2. Considering $\mathbb{Z}$ as a $\mathbb{Z}$-module, where the $\mathbb{Z}$-action on $\mathbb{Z}$ is just multiplication, determine all the $\mathbb{Z}$-submodules of $\mathbb{Z}$.

Solution. Let N be a submodule of $\mathbb{Z}$. Then N is a subgroup of the group $<\mathbb{Z}, +>$. Hence $N = k\mathbb{Z}$ for some $k \in \mathbb{Z}$.

Conversely $k\mathbb{Z}$ is a submodule of $\mathbb{Z}$ for every $k \in \mathbb{Z}$. ∎

February 12, 2004