EXERCISES 2, QUESTION 4

4. Give an example to show that q and r in (2.1.2) are not necessarily unique.

Solution. By Theorem 2.2.3 $\phi_(a + bi) = a^2 + b^2$ is a Euclidean function on $D = \mathbb{Z} + \mathbb{Z}i$. Let $\alpha = 7 + 15i \in D$ and $\beta = 2 \in D$. Then

$$\alpha = \beta\gamma + \delta,\ \phi(\delta) < \phi(\beta),$$

for $(\gamma, \delta) = (3 + 7i, 1 + i)$ and $(3 + 8i, 1 - i)$ showing that the pair (γ, δ) is not unique. ∎

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