

EXERCISES 2, QUESTION 1

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1. Let  $D$  be an integral domain possessing a Euclidean function  $\phi$ . Give an example to show that

$$\phi(a) = \phi(b) \ (a, b \in D) \not\Rightarrow a \sim b.$$

Solution. It is shown in Theorem 2.2.3 that  $\phi(a + bi) = a^2 + b^2$  is a Euclidean function on  $\mathbb{Z} + \mathbb{Z}i$ . Now

$$\begin{aligned}\phi(3 + 4i) &= 3^2 + 4^2 = 9 + 16 = 25, \\ \phi(5) &= 5^2 + 0^2 = 25,\end{aligned}$$

so that

$$\phi(3 + 4i) = \phi(5).$$

However

$$\frac{3 + 4i}{5} = \frac{3}{5} + \frac{4}{5}i \notin \mathbb{Z} + \mathbb{Z}i$$

so that

$$3 + 4i \not\sim 5.$$

Hence  $\phi(a) = \phi(b) \ (a, b \in D) \not\Rightarrow a \sim b$ . ■

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