PROSPECTUS

The text was written primarily to present some new and unusual solutions to the problems and challenges normally encountered during the teaching of Calculus at a higher level, such as College or University. This is not "just another Calculus book"; it is the hope and vision of its author that this book will facilitate the transition from Pre-Calculus to Calculus given the present trends in mathematical education which, in this author's opinion and experience, include inadequate preparation in basic algebra, trigonometry and the occasional difficulty in reading, as a whole.

The basic philosophy underlying its content lies in leaving out complicated looking proofs of theorems and moving these either to the author's Website (a location on the Internet's World Wide Web) or to the Exercises where many hints are given. In this way we may simplify the visual impact of this text and concentrate on those points which one deems important. In the author's opinion, there is a need to review basic material while working towards a common goal which includes the fostering of a feeling for what Calculus is and how to solve problems correctly in the field. The choice and ordering of the subject matter is based upon a purely personal view into the perceived skills acquired by students as they embark upon the first-year experience.

We believe that this text presents an effective solution to the issues outlined above. In the actual classroom settings, where the text has been used for the period 1998-2001, we have found that students get better passing grades when using it and that their ability to develop a conceptual understanding of the basic concepts of Calculus is enhanced greatly. The text has been used by over 4000 students and the comments of all those who chose to submit them are included herein.

At this time a new edition is under preparation, one that will incorporate more traditional and new applications of Calculus. A section on optimization of functions of one or more variables is being prepared along with a section on the traditional related rates. It will be available for distribution as of August, 2001.

Issues and Solutions

Motivation

Every chapter features lots of motivation and is prefaced by a section entitled *The Big Picture*on its aims and uses, historical tidbits and the development of ideas. For example, the chapter on Limits (Chapter 2) includes an opening discussion of the records in the 100 meter Track and Field race and poses the question as to what is the limit of human locomotion? The text is full of effective new strategies for tackling the basic topics in Calculus. The occasional cartoon figure in the margin has helped to facilitate the reading of the material and students have enjoyed their presence.

This is usually followed by a box entitled *Review* where it becomes clear that some skills are more necessary than others for mastery of the subject matter at hand. In some cases we present the material in an odd mix of combined verbal, theoretical, practical, numerical, and geometrical approaches in an attempt to satisfy as many learning styles as possible. The presentation is very personal and it is based upon the author's delivery of the material in a large classroom setting (normally comprised of around 200 students).

At various times in the text, *Shortcuts* are introduced in an attempt to simplify the solution of a given exercise, or class of exercises. Most chapters have individual breaks at a box entitled *Snapshots*. These snapshots consist of *more* examples where we leave out many of the details and outline the rest. We have made a conscious attempt at being repetitive as, in many cases, this is a key to remembering material. Each chapter and its sections concludes with many routine and not so routine exercises that complement the examples. The matter of specific *applications* is treated in a very limited form in this preliminary edition. In many cases, notably in the early chapters, we leave in the most simple of details in order to reinforce those skills which students may find nebulous at times. The student will find it useful to know that the Tables listed under the heading *List of Tables* comprise most of the material and definitions necessary for proficiency in the subject.

Background Preparation

Each chapter includes a section entitled Review where students are referred to background material that is absolutely necessary to the understanding of the material contained therein. The text begins with Chapter 1 which, in itself, is a review of such preliminary material and includes topics such as: The meaning of a function, finding the values of a function, The natural domain of a function, and a quick review of inequalities.

On Functions: Students often find it difficult at interpreting the value of say, $f(2x+1)$ where f is a given function. As a result, a new pedagogical tool called the "Box" method is introduced in Chapter 1 as a solution to the problem of writing down the actual composition of two functions. The Box method has been used with great success in a classroom setting and students have found

it extremely helpful, so helpful in fact, that it makes the task of writing down a composition of two functions an easy task. Such mastery is necessary in the study of Inverse Functions, the Chain Rule, etc. and this technique is used throughout when necessary.

On Inequalities and Absolute Values: In the author's opinion and experience, this is one area where students experience great difficulty. The successful handling of inequalities and inequalities involving absolute values is at the heart of the rigorous approach to Calculus. As a result, a quick yet complete treatment is given in Chapter 1, a presentation which looks at concepts such as "If A ¡ B when is 1/A ¿ 1/B?" or, "What is the square root of the square of a symbol, such as "box", or "1"?". This introduces the absolute value and the whole notion of an absolute value is formulated in terms of boxes, $\square$, so as to gain the most generality. The device of replacing the canonical "x" by a box, 1, is one of the key features of this book. We always emphasize that $x, y, z, \ldots$ are only symbols used as a convenient tool, but actually **any** symbol could have been used in their place (even geometrical shapes).

On Solving Polynomials and Rational Function Inequalities: This also appears to be an area where students find great difficulty. The problem of solving simple quadratic and higher order inequalities is widespread among students entering first year. In most cases in Calculus, the finding of intervals where the graph of a polynomial or rational function is concave up and/or concave down or monotonic requires the solution of some kind of inequality. Our solution lies in a systematic method presented in the chapter on curve sketching, Chapter 5, which enables students to evaluate polynomial and rational function inequalities with some ease. It is closer to the method they have used themselves during their high-school days, but it is more general.

Conceptual Understanding of Ideas

Conscious attempts are made at replacing the symbols x, y, z for universally accepted variables by other symbols like $\heartsuit$, $\diamondsuit$ etc., pointing to the irrelevancy of the "shape" of a symbol in mathematics. Thus, $f(\square) = 2\square+1$ or $f(M) = 2M+1$ is just as good a representation of a function as $f(x) = 2x+1$ or $f(y) = 2y+1$. Furthermore, the Box method is proven to be useful and clarifies the deeper notion of the "concept" of a function. We think of "x", "y", "1", etc. as symbols. Their "shape" in mathematics is not important, but their interaction with other symbols is at the heart of abstraction in mathematics. The more we can relate different kinds of symbols with each other, the better the understanding, as students get a better feel for the *bigger picture*, that of looking at the shape or form of a string of symbols, rather than looking at the specific symbols themselves.

Students often find it difficult to "guess" a limit involving plus or minus infinity. One solution which has proven successful in class can be found in Chapter 2 where we define *extended real numbers* and use them in an extended real number arithmetic in order to help students "guess the right limit" every time! Proving that the limit is indeed the limit is, of course, a different matter and left to another chapter on advanced topics (Chapter 6).

Rolle's Theorem, Bolzano's Theorem and the Mean Value Theorem are presented in a conceptual manner using everyday-life experiences that students will undoubtedly recognize. These results are then stated precisely, but by then students already believe them. Proofs are to be left to the web site or in the exercises for completeness.

The section on inverse functions includes a simple geometrical constructive method for finding the graph of the inverse of a given one-to-one function. This method has already been adopted by some area school mathematics teachers for sketching these graphs. It is based on the fact that reflecting a graph about the line $y = x$ is equivalent to a product of a rotation in the plane and a reflection in three-dimensional space (Chapter 3).

There is a systematic presentation of curve sketching (Chapter 5) starting from the sections on solving polynomial inequalities (rarely done in pre-calculus courses) and then onto solving rational function inequalities (almost never done in pre-calculus courses) and then ending with the "C Me Hava Pizza" rule, a mnemonic device for remembering the main ingredients in sketching any curve. It may sound funny but it is meant to be serious and the reader actually remembers the main steps in graphing a function.

There is less of an emphasis on basic formulae that need to be memorized than on understanding. It is hoped that students will think about what they are learning and remember Calculus, rather than memorize and forget or plug-in and go. This approach has been demonstrated effectively in a classroom setting.

The chapter on Integration (Chapter 7) contains purely conceptual applications of the definite integral to area and volumes of solids of revolution with no need for the derivation of "disc" and "shell" methods that permeate other standard texts. One universal formula for the volume of a simple cylinder and Newton's own idea of *neglecting second order differentials* is all the student needs in order to tackle any volume of a solid of revolution question (regardless of the axis of rotation!).

Repetition and Review

There is a conscious attempt to bridge the "gap" between the pre-calculus experience and the corresponding first year in College vis--vis Calculus. In this spirit, many items from the student's assumed background are deliberately repeated, and recalled (items such as the method of completing the square, basic trigonometric identities, and long division). Indeed, those fundamental trigonometric identities, which are critical in Calculus, are mentioned in the text in many places such as in Chapter 3 and then again in Chapter 8. In fact, the "method of completing the square" is reviewed deliberately just prior to the discussion on trigonometric substitutions in integrals in order to enhance their understanding of the material to come, while the method of long division for factoring polynomials is introduced just before the section on curve sketching for ease of understanding. The use of repetition throughout the text will allow a student to naturally review their work without effort thus acting as an unnoticed classical memory aid.

About Proofs

The subject of proofs will be treated mostly on the World Wide Web or in the exercises so as not to obscure the mostly intuitive approach that is presented in the text. Students tend to fear the abstract proofs presented in most Calculus courses and this appears to be mostly due to a lack of intuition and preparation for the subject (including a lack of expertise in handling basic algebraic manipulations and using elementary trigonometry). Presenting proofs in a more intuitive way has been found to be a more effective means in fostering their understanding of the underlying ideas. Such intuitive descriptions tell students a lot more about the reasoning process in mathematics than they usually get in a standard calculus text, and armed with that reasoning pathway, they can actually construct intuitive proofs themselves, earlier and more easily than before!

A rigorous section on limits is presented in the *Advanced Topics* in order to satisfy the need for rigor in some classroom settings. Rather than sketch some complicated looking picture involving limits and epsilon-boundaries we choose to show that we are really solving some inequalities when we are trying to prove the existence of a limit. The point is that this material is really a consequence of the methods in Chapter 1, namely, knowing **how to solve inequalities and knowing how to manipulate absolute values**. Once again this material is supported by numerical calculations as is the trend these days.

Notes in the margin

These expand and clarify those arguments appearing in the texts. They also include historical subjects, people, hard problems, advanced concepts, general ideas, the uses of calculus etc.

Different Learning Styles

In most instances we try to accommodate as many learning and teaching styles as possible. Thus, some equations are described in words as well as symbols. There is geometric support for many of the statements made in the accompanying text. In this way a student who is visual can immediately refer to the margin for support whereas one who is more intuitive feels at home with the main body of the text. As well, there is accompanying numerical evidence for many of the statements made in the text and shown symbolically. Different teaching styles can be adapted so as to parallel the presentation. The acquisition of knowledge is what we seek on the part of the student and a less formal (teaching) presentation has been shown to be a very effective means of communicating this material in a successful manner.

The language used throughout the text is meant to be easily understood and read by an average student. This is a *deliberate attempt* on the author's part at trying to deliver the material at a very basic level of literacy: This presentation, although unorthodox, has proved to be a success given the wide variety of

students registered in basic Calculus courses.

History

Historical tidbits about mathematicians and others who used mathematics in their work (such as Leonardo da Vinci) are presented here and there to show the interplay between mathematics and other subjects such as the visual arts. Anecdotes about various mathematicians of old are included in order to demystify the belief that mathematicians only dabble in mathematics.

Applications

Many applications are drawn from real-life ordinary experiences (as opposed to research papers). For example, solar flares, the 100-meter dash, the stock exchange, a typical trip to the store, household consumption of electricity, etc. It is important to remember that this text is designed so students can, in the first instance, learn how to solve problems in Calculus. This text is not meant to be a book on Modeling. That approach holds that a student must first learn to read the material, define the variables, relate them together and then finally solve the problem. Placing too much emphasis on applications can have the undesired effect of students saying, "OK, that's interesting but I really don't know how to start it!" This leads to a frustrating situation for the student. By gaining the much-needed confidence in solving problems and *pushing symbols* one can then move on to the problem of correctly reading and interpreting problems of an applied nature.

A Different Approach to Integration

The chapter on integration includes a different and totally justifiable approach to the definite Riemann integral that minimizes the use of Riemann sums (a source of much fear in students). Instead, we define the anti-derivative first, then the definite integral in terms of such anti-derivatives, and finally the definite integral is connected to area using the traditional "Riemann sums" and the Mean Value Theorem.

In this author's opinion, the basic techniques of integration for use by scientists and engineers alike are Integration by substitution, Integration by Parts, Partial Fractions, and Trigonometric Substitutions. One difficulty that constantly arises appears to be in the execution of the method of Integration by Parts. Our solution lies in the introduction of the "Table Method" approach to Integration by Parts appears to be very exciting (and justifiable) and, judging by classroom feedback, extremely effective. The Method outlined in that chapter has been called the "MY CAR" method by my former students. In effect they have created yet another mnemonic device for integrating certain functions by parts, so these devices are in tune with student's learning skills. Throughout the text modifications were introduced with the assistance of students and some of their suggestions were incorporated.

Web References

Most sections include many references to the World Wide Web, references that have been checked by the author, and for the most part, references that are stable. This can allow the student to look up more material on these topics under discussion while they are checking their e-mail ...

Computer Algebra Systems

Although it is not the express purpose of this text at this time to have students solve problems with such systems, we do present material on this on the text's web site. The students can also download a free *MS Windows 3.1* function plotter (from the author's web site) to help in their understanding of curve sketching or to just verify the graphs obtained in the solutions of the exercises in Chapter 5. The plotter can be used as an instrument of mathematical discovery as the student can input any function whose graph is needed, and not only those included in the text. Graphing calculators may be used as well, and problems will be made to address this in a future edition. In this sense we are more conservative than modern texts but then our mission is different, as well.

Exercises and Suggested Homework Problems

In some cases problems and exercises have been assigned a place in the main body of the text. Their purpose is to allow the student a quick try at seeing whether they have understood the material so far. On the other hand, a subsection entitled Suggested Homework Assignments appears at the end of almost every section thereby relieving the Instructor of the task of choosing suitable problems for homework. Our experience indicates that students enjoy tackling these suggested problems even when they are not assigned!

Customized Texts

Answers to all the questions in the Exercises are usually included with the text but need not be so, as the text may be customized depending on individual course requirements. In other words, a text can be created for the express purpose of the course outline of Calculus X at University Y. As well, the web site will include the some proofs of the basic theorems and results on Calculus and links will be incorporated into the text as to their location on the Web.

All the answers (some very detailed) to the questions on Integration are included for reference purposes into the text itself. Continuous classroom experience has strengthened the belief that students are much more comfortable with a display of all the solutions in the text rather than a selected few (or only the even numbers with no justification). In our opinion, students really want detailed solutions and they will actually learn from them. But these solutions must be accessible, either on the Web or in the text.

The Web Site

The web site is to be coded in the old standard, - HTML 2.0, for universality. We are assuming a worldwide baseline standard of 386 PC's running at 8 MHz and using Windows 3.1. Such a machine cannot successfully run JAVA scripts or Windows 95 due to hardware conflicts. Thus, our vision is a web site that is accessible to almost anyone who can use the Internet. There the students will find proofs of the basic results mentioned in the text; interactive online server graded exercises, sample quizzes, tests and final examinations. The site is meant to go hand-in-hand with the text.