Chapter 6, Question 4

4. Let m be a squarefree integer. Let $K = \mathbb{Q}(\sqrt{m})$. Prove that

$$\sigma_1(x + y\sqrt{m}) = x + y\sqrt{m} \ (x, y \in \mathbb{Q})$$

and

$$\sigma_2(x + y\sqrt{m}) = x - y\sqrt{m} \ (x, y \in \mathbb{Q})$$

are the only monomorphisms from K to $\mathbb{C}$.

Solution. As m is a squarefree integer, $[\mathbb{Q}(\sqrt{m}) : \mathbb{Q}] = 2$ (Theorem 5.4.1). Hence there are exactly two monomorphisms from $K = \mathbb{Q}(\sqrt{m})$ to $\mathbb{C}$ (Theorem 6.2.1). But σ_1 and σ_2 are easily checked to be monomorphisms : $K \to \mathbb{C}$. Clearly $\sigma_1 \neq \sigma_2$. Thus σ_1 and σ_2 are the only monomorphisms : $K \to \mathbb{C}$. ∎

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