Dynamic Quantizer Design for Hidden Markov State Estimation Via Multiple Sensors With Fusion Center Feedback

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Abstract—**This paper considers the state estimation of hidden Markov models by sensor networks. The objective is to minimize the long term average of the mean square estimation error for the underlying finite state Markov chain. By employing feedback from the fusion center, a dynamic quantization scheme for the sensor nodes is proposed and analyzed by a stochastic control approach. Dynamic rate allocation is also considered when the sensor nodes generate mode dependent measurements.**

Index Terms—**Dynamic programming equation, dynamic quantization, hidden Markov models, sensor networks, state estimation.**

I. Introduction

SENSOR networks have gained intensive research interest due to their wide range of current and potential applications in environment surveillance, detection and estimation, and location awareness services, etc. [7]. While completely distributed computation is possible in sensor networks where sensors communicate with each other [20], there is an alternative framework that has been well researched and is adopted as the sensor network infrastructure in this paper. In such networks, geographically distributed sensors send data to a fusion center (FC) (rather than communicating with each other), which is assumed to have a higher computation capability than the sensors themselves. Due to their limited on board battery power the sensors not only have little computational capacity but also possess limited communication capability as data processing and transmission both require energy, the energy required for data transmission usually being the dominant component. The channel between each sensor and the fusion center is usually bandwidth limited (e.g., a wireless link), and hence only a quantized output can be transmitted where the number of quantization levels is limited by the data rate constraints of the channel. The fusion center combines the data received from all sensors to make a decision or form an estimate of the observed process. In general, the information that can be obtained from a single sensor is limited. However, when a set of sensors is used, it is possible to achieve a satisfactory performance [28], [6].

Sensor networks have been studied in different contexts according to their application backgrounds. Within the context of statistical signal processing, an important application of sensor networks is state estimation of random processes, since in reality sensor networks operate in a time-varying environment and the resulting sensor measurements provide partial information only of such random processes usually modeled by dynamical systems [11]. For instance, the sensors may have noisy observation of a moving target in a battlefield, or changing temperature in a bushfire prone area, and so on. In certain applications of interest, the underlying random process may be modeled as a Markov chain (e.g., the maneuvering modes of a target, the protein levels in blood cells, etc.) and the resulting measurements modeled by hidden Markov chains. (See [26] for near-optimal quantizer design for hidden binary Markov chains for a single sensor, and for an extension to the two-sensor case, see [8]. Also, see [13] for a multiple sensor scheduling solution using hidden Markov models. Interested readers are also referred to [5], which deals with reconstructing centralized optimal estimates at the fusion center for Markov processes using locally sufficient statistics calculated by distributed stations. However, data rate constraints are not considered in [5]).

In general the design of optimal quantizer problem for sensor networks is not a trivial task even when the Markov chain has only a few states. This may be attributed to the high complexity in the associated nonconvex optimization problems. In the context of Neyman–Pearson distributed detection, the problem of locally optimal threshold selection for the sensor quantizers is addressed in [22]. For the distributed estimation of a single random variable, the Lloyd-Max algorithm may be employed for optimization of the quantizer with local optimality [12]. See also [26] and [8] for quantizer design algorithms with information theoretic optimality criteria.

In this paper we consider the estimation of finite state Markov chains via sensor networks where sensors send quantized measurements to a fusion center. For computational tractability, we start by analyzing binary quantization at the sensor nodes and later consider sensors with variable rates where some sensors may be allowed to have higher rates than one bit per symbol. In general, a binary quantization scheme can only transmit very coarse information, and traditionally the network performance is improved by increasing the number of sensors. There has been an extensive literature on binary sensors in the context of

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hypothesis testing (see [6], [16], [23], and references therein). Recent applications of binary sensors for target tracking can be found in [3] and [19]. In fact, binary sensors are useful for tracking partial motion information such as directional information of a moving object, or the change trend (increase or decrease) of certain natural phenomena (see, e.g., [3]). (For optimum local decision space partitioning for multibit decision values, see [15].)

In this paper, instead of improving the estimation by increasing the number of sensors, we will adopt another approach by establishing a feedback scheme from the fusion center to the sensors so that a certain coordination of the sensors may be maintained. The consequence of the feedback is that the usual static quantization scheme is then replaced by a dynamic one. Concerning the communication and computational capability in such a sensor network, we make a few basic assumptions. First, we assume that the quantized output at each sensor node can be sent to the fusion center without any error. In practice, such error-free transmission of a finite symbol set over a wireless channel may be achieved with use of appropriate error control coding schemes. Second, we do not impose any constraint on the computational capability of the fusion center. We also assume that the feedback channels between the fusion center and the sensors allow error-free transmission of the computed quantizer parameters. Notice that in such a network infrastructure, there is no direct communication between any two-sensor nodes. Evidently, in this paper the communication pattern between the fusion center and the sensors is more complicated compared to unidirectional sensor networks. However, this approach has the potential to reduce the network complexity from another point of view, i.e., in order to achieve a prescribed performance, one only needs to implement fewer sensor nodes compared to the case without feedback. This kind of feedback information pattern has been employed for performance improvement in the sensor network literature, but mainly in the context of hypothesis testing [21], [1], [25], [27], and is referred to as decision feedback. There is an underlying assumption here that the fusion center has sufficient power at its disposal in contrast to the sensor nodes which have limited energy that cannot perhaps be replenished. This allows us to employ dynamic feedback from the fusion center and reduce the number of static sensors (with static quantizers without dynamic feedback from the fusion center) required to achieve a comparable performance. This clearly results in less total energy consumption at the sensor nodes and also reduced bandwidth demand on the multiaccess wireless network in communicating information to the fusion center.

The main contributions of this paper can be summarized as follows.

1) We develop a conceptual framework for dynamic optimization for quantizer design using a stochastic control framework for state estimation of Markov sources observed through noisy quantized measurements via a network of sensors. We believe that this paper is the first attempt in considering dynamic optimal quantizer design using a stochastic control approach for the purpose of state estimation for hidden Markov models. We further show that there is a systematic way to construct a numerical method for computation of the quantization scheme, which avoids sample path dependent simulations.
2) Due to the nonconvex nature of the above optimization problem, we design a suboptimal algorithm based on a Markov decision process (MDP) approach where the quantizer thresholds belong to a finite set of discrete values. A relative value iteration method is then applied to solve a discretized version of the Bellman equation for the original problem where quantizer thresholds are allowed to assume any real value.
3) In addition, we consider situations where the multiple sensor measurements may be mode dependent, e.g., dependent on the location of a moving object or actions of a maneuvering target. These situations demand that different sensors are allocated different data rates with a constraint on the total data rate, such that sensor measurements containing more information are given higher resolution. Using a stochastic control framework, we design and analyze a novel efficient integrated dynamic quantization and data rate allocation algorithm for multiple sensors providing mode dependent noisy quantized measurements.
4) Finally, through computer simulation studies, we illustrate the superior performance of our algorithms implementing dynamic quantization over those with static quantizers.

The rest of paper is organized as follows. Section II formulates the state estimation problem within the framework of hidden Markov models with quantized output. In Section III, an equivalent completely observable stochastic control problem is formulated by applying the so-called information state based approach. The dynamic programming equation for the resulting stochastic control problem is studied in Section III. In Section IV a joint dynamic rate allocation and quantizer design method is analyzed for a mode dependent model as motivated by various application scenarios such as tracking maneuvering targets by multiple sensors. Section V presents numerical results and illustrates the performance improvement achieved by our novel dynamic quantization algorithm. Section VI concludes the paper.

II. System Model

Let $\{x_t, t \geq 1\}$ be a discrete time Markov chain with state space $S = \{s_1, \ldots, s_n\}$ and transition matrix

$$\mathbf{P} = \begin{bmatrix} p_{11} & \cdots & p_{1n} \\ \vdots & & \vdots \\ p_{n1} & \cdots & p_{nn} \end{bmatrix}$$

where $p_{ij} = P(x_{t+1} = s_j \,|\, x_t = s_i), 1 \leq i, j \leq n$. Assume without loss of generality that $s_1 < \cdots < s_n$. Let the measurement of the M sensors be specified by

$$y_{m,t} = x_t + w_{m,t} \quad 1 \leq m \leq M. \tag{2.1}$$

A similar model for a two state Markov chain with one sensor has been studied in [26] and performance analysis is based upon static quantization with different quantization levels. Write (2.1) in the vector form

$$\mathbf{y}_t = \mathbf{a}x_t + \mathbf{w}_t \tag{2.2}$$

where $\mathbf{y}_t = [y_{1,t}, \ldots, y_{M,t}]^T, \mathbf{a} = [1, \ldots, 1]^T$, and $\mathbf{w}_t = [w_{1,t}, \ldots, w_{M,t}]^T$. The noise $\{\mathbf{w}_t, t \geq 1\}$ is a sequence of independent and identically distributed (i.i.d.) vector random variables.

For a set of M binary sensors, any given quantization scheme is specified by M sequences of constants $\{r_{m,t}, t \geq 1\}, 1 \leq m \leq M$, where $r_{m,t}$ is used to partition the range space of $y_{m,t}$ measured by the mth sensor node. Let $\mathbf{r}_t = (r_{1,t}, \ldots, r_{M,t})$, and write the quantization sequence $\{\mathbf{r}_t, t \geq 1\} = \{(r_{1,t}, \ldots, r_{M,t}), t \geq 1\}$. At time t, let the data (also to be called message) that the fusion center receives from the mth sensor be denoted by $y^q_{m,t}$. One may take any two distinct symbols a_1 and a_2 such that the events $\{y_{m,t} < r_{m,t}\}$ and $\{y_{m,t} \geq r_{m,t}\}$ are equivalent to $\{y^q_{m,t} = a_1\}$ and $\{y^q_{m,t} = a_2\}$, respectively. Hence, the received message at the fusion center is

$$y^q_{m,t} = \begin{cases} a_1, & y_{m,t} < r_{m,t} \\ a_2, & y_{m,t} \geq r_{m,t}. \end{cases} \tag{2.3}$$

Let $\mathbf{y}^q_t = [y^q_{1,t}, \ldots, y^q_{M,t}]^T$ and denote $\mathbf{y}^q_t$ by

$$\mathbf{y}^q_t = \mathcal{Q}(\mathbf{r}_t, y_{1,t}, \ldots, y_{M,t}) \tag{2.4}$$

where the map $\mathcal{Q} : \mathbb{R}^M \times \mathbb{R}^M \to \{a_1, a_2\}^M$ is determined from (2.3) in an obvious manner. Here $\{a_1, a_2\}^M$ denotes the M-fold Cartesian product of the set $\{a_1, a_2\}$, the common code book for all sensor nodes.

The objective of this paper is to dynamically obtain $\{\mathbf{r}_t, t \geq 1\}$ by optimizing an appropriate cost criterion at the fusion center, based on the measurements $(y^q_{1,i}, \ldots, y^q_{M,i}), i \leq t-1$ received from the M sensors. The optimal quantization levels are then sent back to the sensors via feedback channels to be used for quantizing the measurements at the next time instant t. This scheme is shown in Fig. 1 where all the relevant communication flows are labelled. In the next section, we formulate this problem formally as a stochastic optimal control problem.

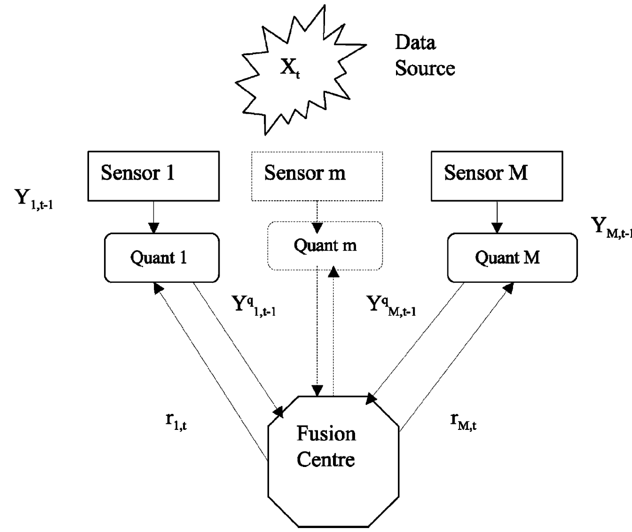

Fig. 1. Dynamic quantization scheme with feedback from fusion center.

III. Bellman Dynamic Sensor Optimization

The dynamic quantization problem may be regarded as a generalized stochastic control problem in which $\mathbf{r}_t$ affects the observation $\mathbf{y}^q_t$ at the fusion center, but the state variable x_t is autonomous. Since the fusion center is generally equipped with a high computational capability and data storage capacity, we assume the parameters $\mathbf{r}_t$, $t \geq 1$, are computed at the fusion center as a function of $(\mathbf{y}^q_1, \mathbf{r}_1, \ldots, \mathbf{y}^q_{t-1}, \mathbf{r}_{t-1})$. In other words, $\mathbf{r}_t$ is adapted to $\mathcal{F}_{t-1} \triangleq \mathcal{F}(\mathbf{y}^q_i, \mathbf{r}_i, i \leq t-1)$ which is the σ-algebra generated by the past observations and controls. In further analysis, a recursively calculated sufficient statistic shall be identified such that $\mathbf{r}_t$ need not be determined using the overall history $(\mathbf{y}^q_1, \mathbf{r}_1, \ldots, \mathbf{y}^q_{t-1}, \mathbf{r}_{t-1})$ when the sufficient statistic is computed at each step. Once $\mathbf{r}_t$ is computed, the entry $r_{m,t}$ is sent from the fusion center to the mth sensor. Here, to ensure causality, it is important to require that $\mathbf{r}_t$ depends on the quantized measurements up until time $t-1$ so that it can be determined in the epoch between $t-1$ and t. In this framework, the decentralized nature of the network is preserved in the sense that the data is preprocessed at the sensor node level based upon which the fusion center forms a final estimate, and no direct communication exists between the sensors except that each sensor receives feedback commands from the fusion center.

Define the so-called information state [14]

$$\theta_t = [\theta_{1,t}, \ldots, \theta_{n,t}]^T$$
$$\text{where} \quad \theta_{i,t} = P[x_t = s_i | \mathcal{F}_t], \quad 1 \leq i \leq n, \quad t \geq 1.$$

The component $\theta_{i,t}$ provides a measure of likelihood of x_t staying at the state s_i given the measurements $(\mathbf{y}^q_i, \ldots, \mathbf{y}^q_t)$. By the Bayesian rule, θ_t is recursively given as

$$\theta_{t+1} = \frac{1}{\varphi_{t+1}} \mathbf{Q}\left(s_1, \ldots, s_n, \mathbf{r}_{t+1}, \mathbf{y}^q_{t+1}\right) \mathbf{P}^T \theta_t \tag{3.1}$$

where $\mathbf{P}$ is the transition probability matrix of x_t, φ_{t+1} is a normalizing factor such that $\sum_i \theta_{i,t+1} = 1$, and

$$\mathbf{Q}\left(s_1, \ldots, s_n, \mathbf{r}_t, \mathbf{y}^q_t\right) = \mathrm{Diag}\left[F\left(s_1, \mathbf{r}_t, \mathbf{y}^q_t\right), \ldots, F(s_n, \mathbf{r}_t, \mathbf{y}^q_t)\right]_{n \times n}. \tag{3.2}$$

Note that

$$F\left(s_i, \mathbf{r}_t, (a_{i_1}, \ldots, a_{i_M})\right) = \int_{\mathcal{A}(\mathbf{r}_t, (a_{i_1}, \ldots, a_{i_M}))} f(y_1 - s_i, \ldots, y_M - s_i) dy_1 \cdots dy_M \tag{3.3}$$

where $\mathcal{A}(\mathbf{r}_t, (a_{i_1}, \ldots, a_{i_M})) \triangleq \{y \in \mathbb{R}^M, \mathcal{Q}(\mathbf{r}_t, y) = (a_{i_1}, \ldots, a_{i_M})\}, i_l \in \{1, 2\}$ for $l = 1, 2, \ldots, M, f$ is the joint probability density for $\mathbf{w} = (w_{1,t}, \ldots, w_{M,t})^T$, and $\mathcal{Q}$ is defined in (2.4). For example, in the special case of two sensors, $(M = 2)$, then

$$F(s_i, \mathbf{r}(a_1, a_1)) = \int_{-\infty}^{r_1} \int_{-\infty}^{r_2} f(y_1 - s_i, y_2 - s_i) dy_1 dy_2$$

etc., where (a_1, a_1) corresponds to a specific outcome of $\mathbf{y}_t^q$ and determines a specific integration region.

Given $\mathcal{F}_t$, the conditional expectation of x_t is

$$\hat{x}_t = E[x_t \,|\, \mathcal{F}_t] = \sum_{i=1}^{n} s_i \theta_{i,t}. \tag{3.4}$$

In fact, for any given quantization sequence $\{\mathbf{r}_t, t \geq 1\}$,

$$E|x_t - \hat{x}_t|^2 = \inf_{\xi_t} E|x_t - \xi_t|^2$$

where ξ_t is any random variable adapted to $\mathcal{F}_t$.

For each sequence $\{\mathbf{r}_t, t \geq 1\}$, the long term average of the mean square error for the state estimation is given as

$$J(\mathbf{r}) = \limsup_{N\to\infty} \frac{1}{N} \sum_{t=1}^{N} E|x_t - \hat{x}_t|^2 \tag{3.5}$$

where the sequence $\{\mathbf{r}_t, t \geq 1\}$ is simply indicated as $\mathbf{r}$ and $\hat{x}_t$ is taken as the conditional expectation in (3.4). In subsequent analysis we may also use $\mathbf{r}$ to denote a vector in $\mathbb{R}^M$. In this paper, we use the l_1 norm $\|\mathbf{z}\|_1 \triangleq \sum_{i=1}^n |z_i|$ for $\mathbf{z} \in \mathbb{R}^n$.

Now, define the conditional cost

$$c(\theta_t) = E[|x_t - \hat{x}_t|^2 | \mathcal{F}_t] = \sum_{i=1}^{n} \left[s_i - \sum_{j=1}^{n} s_j \theta_{j,t} \right]^2 \theta_{i,t}$$

which is computed by (3.4). In the special case of $n = 2, c(\theta_t)|_{n=2} = (s_1 - s_2)^2 \theta_{1,t} \theta_{2,t}$.

The optimal estimation problem associated with (3.5) may be equivalently expressed as

$$(\mathbb{P}) \quad \text{minimize} \quad J(\mathbf{r}, \theta) = \limsup_{N\to\infty} \frac{1}{N} \sum_{t=1}^{N} E[c(\theta_t) \,|\, \theta_1 = \theta] \tag{3.6}$$

where θ is the initial condition and $\mathbf{r}$ denotes the quantization sequence $\{\mathbf{r}_t, t \geq 1\}$ such that each $\mathbf{r}_t$ is adapted to $\mathcal{F}_{t-1}$. Notice that the fusion center cannot directly minimize the cost (3.5) since it does not have exact knowledge of x_t. However, it can solve the problem $(\mathbb{P})$ since θ_t may be recursively computed using $\mathbf{y}_i^q, i \leq t$. Indeed, $(\mathbb{P})$ is an infinite horizon average cost based stochastic control problem, and its associated dynamic programming (Bellman) equation is given as

$$\begin{aligned} &\lambda + h(\theta) \\ &= \min_{\mathbf{r}} \left[c(\theta) + \sum_{\mathbf{y}^q} \|\mathbf{T}(\mathbf{r}, \mathbf{y}^q)\theta\|_1 \times h\left(\frac{\mathbf{T}(\mathbf{r}, \mathbf{y}^q)\theta}{\|\mathbf{T}(\mathbf{r}, \mathbf{y}^q)\theta\|_1} \right) \right] \\ &\triangleq \min_{\mathbf{r}} \Phi(\theta, \mathbf{r}) \end{aligned} \tag{3.7}$$

where $\mathbf{y}^q \in \{a_1, a_2\}^M$ and $\mathbf{T}(\mathbf{r}_t, \mathbf{y}_t^q) = \mathbf{Q}(s_1, \ldots, s_n, \mathbf{r}_t, \mathbf{y}_t^q)\mathbf{P}^T$. The function $h(\theta)$ will be called as the differential cost. Define the simplex

$$S_1 \triangleq \{\alpha \in \mathbb{R}_+^n, \|\alpha\|_1 = 1\}.$$

Theorem 3.1: Assume there exist $\lambda \in \mathbb{R}$ and a bounded function $h : S_1 \to \mathbb{R}$, satisfying (3.7), and there is a measurable function $\mathbf{r} = g(\theta), \theta \in S_1$, such that $g(\theta) = \arg\min_{\mathbf{r}} \Phi(\theta, \mathbf{r})$. Then the quantization with $\mathbf{r}_t = g(\theta_{t-1}), t \geq 2$, minimizes the cost in (3.6) with the optimal cost λ. □

Remark: The theorem is essentially an adaptation of the standard verification theorem for optimal Markov decision problems with Borel state spaces. Notice that in Theorem 3.1, $\mathbf{r}_1 \in \mathbb{R}^M$ may be chosen as any fixed constant and it does not affect the optimal cost. Existence of a solution to (3.7) is insured based upon mild conditions in terms of its exponentially discounted version; (see, e.g., [9]). □

In the case the quantization is static, the resulting cost λ_0 may be specified as follows:

$$\lambda_0 + h_0(\theta) = c(\theta) + \sum_{\mathbf{y}^q} \|\mathbf{T}(\mathbf{r}, \mathbf{y}^q)\theta\|_1 h_0 \left(\frac{\mathbf{T}(\mathbf{r}, \mathbf{y}^q)\theta}{\|\mathbf{T}(\mathbf{r}, \mathbf{y}^q)\theta\|_1} \right) \tag{3.8}$$

with $\mathbf{r} = (r_1, \ldots, r_M)$, which is of a degenerate form of (3.7) since the domain for $\mathbf{r}$ is now a singleton. Equation (3.8) is useful for the performance calculation of any static binary quantizer.

Remark: Theorem 3.1 and the associated numerical scheme in the following Section III may be easily generalized to the case of more than two quantization levels. We will not repeat the details.

From a numerical computational point of view, a solution to (3.7), if existing, is difficult to solve since for a fixed θ, the right-hand side of (3.7) is a nonconvex function of the continuous variable $\mathbf{r} \in \mathbb{R}^M$. For numerical tractability, instead of achieving an arbitrary approximation to the optimal solution, in this section a variant of the problem $(\mathbb{P})$ is considered where $\mathbf{r}$ is restricted to a finite setof discrete values. The following steps are carried out.

a) Choose a finite subset in $\mathbb{R}^M$ as the range space of $\mathbf{r}$.
b) As a suboptimal approximation to $(\mathbb{P})$, discretize the information state and derive a finite dimensional equation which, in fact, corresponds to a well defined optimal Markov decision problem with finite states and finite control actions. Such a discretized Bellman equation is intended to specify a suboptimal solution to the problem $(\mathbb{P})$.
c) Solve the fully discretized Bellman equation by the *relative value iteration* algorithm [4].

For notational and computational simplicity, the same finite subset of $\mathbb{R}$ is employed for optimizing each entry r_m in $\mathbf{r} \in \mathbb{R}^M, 1 \leq m \leq M$. Now, let the range space of $r_{m,t}$ be $L_d =$

$\{\gamma_1, \dots, \gamma_d\} \subset \mathbb{R}$. Hence, $\mathbf{r}$ shall be chosen from the product set L_d^M. Write the corresponding Bellman equation as

$$\lambda + h(\theta) = \min_{\mathbf{r} \in L_d^M} \left[c(\theta) + \sum_{\mathbf{y}^q} \|\mathbf{T}(\mathbf{r}, \mathbf{y}^q)\theta\|_1 h\left(\frac{\mathbf{T}(\mathbf{r}, \mathbf{y}^q)\theta}{\|\mathbf{T}(\mathbf{r}, \mathbf{y}^q)\theta\|_1} \right) \right] \tag{3.9}$$

We will establish the solvability of the Bellman equation (3.9). Let us introduce the following assumption.

H1): For any $\mathbf{r} \in L_d^M$ and $\mathbf{y}^q \in \{a_1, a_2\}^M$, the matrix $\mathbf{T}(\mathbf{r}, \mathbf{y}^q)$ is nonsingular and strictly positive. □

Notice that H1) holds under very mild conditions for the noise. For the example of $M = 2$ for two sensors, H1) holds for nonsingular and positive $\mathbf{P}$ and any i.i.d. noise sequence $\{\mathbf{w}_t, t \geq 1\}$ such that for any $C > 0$, each of the four events $\{\mathbf{w}_t \in I_{-C} \times I_{-C}\}, \{\mathbf{w}_t \in I_{-C} \times I_C\}, \{\mathbf{w}_t \in I_C \times I_{-C}\}, \{\mathbf{w}_t \in I_C \times I_C\}$, $t \geq 1$ has strictly positive probability, where $I_{-C} = (-\infty, -C]$ and $I_C = [C, \infty)$. This criterion is obviously satisfied for any nondegenerate bivariate Gaussian noise distribution.

Proposition 3.2: Under H1), there exist λ and a bounded function h satisfying (3.9).

Proof: See the Appendix. □

Remark: By the verification theorem [9], the constant λ specified in Proposition 3.2 may be interpreted as the minimum for $J(\mathbf{r})$ in (3.5) when each $\mathbf{r}_t$ is restricted to be in L_d^M and adapted to $\mathcal{F}_{t-1}$ (i.e., it depends on $\mathbf{y}_1^q, \dots, \mathbf{y}_{t-1}^q$). □

In Proposition 3.2, H1) may be relaxed such that $\mathbf{T}(\mathbf{r}, \mathbf{y}^q)$ is only primitive (see, e.g., [24]) and nonsingular [9] for any $\mathbf{r} \in L_d^M$ and $\mathbf{y}^q \in \{a_1, a_2\}^M$. The details are omitted here.

Now we devise a scheme for a numerical solution to (3.9). For notational simplicity, the numerical procedure for solving (3.9) is described for the case of $n = 2$, i.e., $\theta \in \mathbb{R}^2$. The same procedure can be employed for the case $n > 2$. Taking $n = 2$, let the range space S_1 of θ be discretized with a step size $1/N$. Let $S_{1,N} = \{[k/N, 1 - k/N]^T, k = 0, \dots, N\}$. Take $\theta \in S_{1,N}$ for the left-hand side of (3.9). However, due to the linear transformation and normalization inside the function h, the right-hand side of (3.9) involves values of h at points outside $S_{1,N}$. Hence, this cannot induce an equation only in terms of values of h on the grid $S_{1,N}$. To overcome this difficulty, we consider an approximation by rounding off $\theta' = \mathbf{T}\theta/\|\mathbf{T}\theta\|_1$ to the closest point θ'' in $S_{1,N}$, and then we simply replace $h(\theta')$ by $h(\theta'')$. This procedure leads to a fully discretized equation, as follows:

$$\lambda + h(l_k) = \min_{\mathbf{r} \in L_d^M} \left[c(l_k) + \sum_{\mathbf{y}^q} \|\mathbf{T}(\mathbf{r}, \mathbf{y}^q) l_k\|_1 \times h\left(\left[\frac{\mathbf{T}(\mathbf{r}, \mathbf{y}^q) l_k}{\|\mathbf{T}(\mathbf{r}, \mathbf{y}^q) l_k\|_1} \right]_{\text{round}} \right) \right] \tag{3.10}$$

where $l_k \in S_{1,N}$ represents a discretized approximation of θ, and for $\theta = [\beta_1, \beta_2]^T \in S_1, [\beta]_{\text{round}} = ([\beta_1]_{\text{round}}, 1 - [\beta_1]_{\text{round}})^T$ with

$$[\beta_1]_{\text{round}} = \begin{cases} \frac{k}{N}, & \text{for } \beta \in \left(\frac{k}{N} - \frac{1}{2N}, \frac{k}{N} + \frac{1}{2N} \right] \\ 0, & \text{for } \beta \in \left[0, \frac{1}{2N} \right] \\ 1, & \text{for } \beta \in \left(1 - \frac{1}{2N}, 1 \right]. \end{cases}$$

Thus, the continuum information state $\theta \in S_1$ is approximated by a certain point $l_k \in S_{1,N}$.

Remark: Note that the exact choice of the discretization step size $1/N$ is rather empirical. In general, it is difficult to get an explicit relationship between the approximation error (due to discretization) and the step size. Here, we choose $1/N$ to be much smaller compared to the magnitude of h and λ. □

The following proposition is useful for guaranteeing convergence of the iterative discretized numerical scheme to be introduced later.

Proposition 3.3: For any $\theta \in S_1$, we have

$$\sum_{\mathbf{y}^q} \|\mathbf{T}(\mathbf{r}, \mathbf{y}^q)\theta\|_1 = 1. \tag{3.11}$$

Proof: See the Appendix. □

We observe that for a fixed $l_k \in S_{1,N}$ the summation on the right-hand side of (3.10) involves the value of h at 2^M points $l_k^{(i)}, 1 \leq i \leq 2^M$ (derived from the rounding off procedure with different values for $\mathbf{y}^q$), each associated with a weight coefficient $\delta_k^{(i)}$, depending on $\mathbf{y}^q$ and satisfying $\sum_{i=1}^{2^M} \delta_k^{(i)} = 1$ by Proposition 3.3. Hence, (3.10) is equivalent to the Bellman equation for a standard finite state Markov decision problem where $\delta_k^{(i)}, 1 \leq i \leq 2^M$ constitute the nonzero entries in the kth row of the $(N+1) \times (N+1)$ dimensional controlled transition probability matrix. Notice that the equivalent Markov decision problem has the state space $S_{1,N}$. And therefore, (3.10) can be solved by the relative value iteration method which converges to its exact solution (see [4] for details).

In summary, the fusion center, after obtaining $\mathbf{y}_i^q, i \leq t$, computes the information state θ_t and then translates it into a value for $\mathbf{r}_{t+1}$ which has been computed offline as a function of θ_t using the relative value iteration algorithm implemented for the discretized Bellman equation (3.10). These new quantization threshold values are then communicated back to the sensors.

IV. Generalization to Variable Sensor Rates

In this section, we consider situations where the observations of the sensor nodes are mode dependent. This mode could represent a specific location of a moving object or a maneuvering mode of a moving target. Within this modeling paradigm it is of interest to consider dynamic (variable) rate allocation under the condition that the total rate of the sensors is constrained due to the shared communication channel. The intuitive justification of dynamic rate allocation with the mode dependent observations is that at time t, if it is inferred by the fusion center from posterior information that the system is more likely to be operating in mode $x_t = s_{m_i}$ for which a certain sensor has a higher measurement gain, then that sensor should be assigned more rate for refined estimates, and that the consequently reduced rates for other sensors should result in far less performance loss since their observations are less useful due to their low signal to noise ratios. The main idea for dynamic rate allocation is that one may choose the quantization parameters $\mathbf{r}_t, t \geq 1$ such that the corresponding partition at the sensors does not produce a total rate (or total number of quantization levels) exceeding a specified number, and it is allowed to split the number of quantization levels unevenly at the sensors.

Motivated by the above, we consider the following (somewhat simplified) system model:

$$y_{m,t} = g_m(x_t)z_t + w_{m,t}, \quad 1 \leq m \leq M \tag{4.1}$$

where

$$P\left(z_{t+1} = s_j^z \,\middle|\, z_t = s_i^z, x_t = s_k\right) = p_{ij,k}^z$$

for which z_t has state space $S^z = \{s_1^z, \ldots, s_n^z\}$. The i.i.d. noise sequence $\{\mathbf{w}_t, t \geq 1\}$ has a probability density f. Indeed, the above modeling of z_t may be regarded as a simplified discrete approximation of the hybrid continuum modeling of the target state in the tracking literature (see [17] and [18]). Here, x_t models the multiple modes which affect the measurement SNR.

For notational simplicity, in the following formulation a system of two sensors is analyzed where x_t and z_t have state space $S = \{s_1, s_2\}$ and $S^z = \{s_1^z, s_2^z\}$, respectively. The generalization to the case of more states is obvious. Denote the transition matrix of x_t by $\mathbf{P} = (p_{ij})_{1\leq i,j\leq 2}$, and let the transition matrix of z_t given $x_t = s_i$ be given as $\mathbf{P}^z|_{x_t=s_1} = (p_{ij}^z)_{1\leq i,j\leq 2}$, $\mathbf{P}^z|_{x_t=s_2} = (\hat{p}_{ij}^z)_{1\leq i,j\leq 2}$.

The quantization scheme consists of one binary sensor with symbol set $\{a_1, a_2\}$, and a ternary one with symbol set $\{a_1, a_2, a_3\}$. Hence, the total number of quantization levels is 5. Furthermore, let the parameter r_{bin} for the binary sensor be chosen from the set $L_{d_1}^b = \{\gamma_1, \ldots, \gamma_{d_1}\}$. The ternary quantizer is specified by a pair r_{ter} in the set $L_{d_2}^t = \{(\gamma_1^i, \gamma_2^i), 1 \leq i \leq d_2\}$. Now any quantizer, denoted simply as $\mathbf{r}_t$, at time t may be represented as $(\gamma_1; \gamma_2, \gamma_3)$ with the first sensor being binary, or $(\gamma_1, \gamma_2; \gamma_3)$ with the second being binary. Hence, an admissible quantizer $\mathbf{r}_t$ is an entry in the union $\mathcal{L} \triangleq (L_{d_1}^b \times L_{d_2}^t) \cup (L_{d_2}^t \times L_{d_1}^b)$ of two sets, each being an ordered Cartesian product. For instance, $(\gamma_1, \gamma_2; \gamma_3)$ is in $L_{d_2}^t \times L_{d_1}^b$ where $(\gamma_1, \gamma_2) \in L_{d_2}^t$ and $\gamma_3 \in L_{d_1}^b$. Once $\mathbf{r}_t$ is selected, the message $\mathbf{y}_t^q = [y_{1,t}^q, y_{2,t}^q]^T$ received by the fusion center is an entry in $(\{a_1, a_2\} \times \{a_1, a_2, a_3\}) \cup (\{a_1, a_2, a_3\} \times \{a_1, a_2\})$. As in Section II, denote the quantizer output by $\mathbf{y}_t^q = \mathcal{Q}(\mathbf{r}_t, \mathbf{y}_t)$.

For the estimation of (x_t, z_t), the cost is specified by the weighted mean-square error

$$J(\mathbf{r}) = \limsup_{N\to\infty} \frac{1}{N} \sum_{t=1}^{N} [\beta E|x_t - \hat{x}_t|^2 + E|z_t - \hat{z}_t|^2]$$

where $\beta > 0$. Define the information state $\theta_t = [I_{11}, I_{12}, I_{21}, I_{22}]^T$ where $I_{ij}(t) = E[x_t = s_i, z_t = s_j^z \,|\, \mathbf{y}_1^q, \ldots, \mathbf{y}_t^q, \mathbf{r}_1, \ldots, \mathbf{r}_t]$. Here, $\mathbf{y}_t^q$ denotes the quantized output of the sensors. Define

$$\mathbf{D} = \begin{pmatrix} p_{11}p_{11}^z & p_{11}p_{12}^z & p_{12}p_{11}^z & p_{12}p_{12}^z \\ p_{11}p_{21}^z & p_{11}p_{22}^z & p_{12}p_{21}^z & p_{12}p_{22}^z \\ p_{21}\hat{p}_{11}^z & p_{21}\hat{p}_{12}^z & p_{22}\hat{p}_{11}^z & p_{22}\hat{p}_{12}^z \\ p_{21}\hat{p}_{21}^z & p_{21}\hat{p}_{22}^z & p_{22}\hat{p}_{21}^z & p_{22}\hat{p}_{22}^z \end{pmatrix}$$

which is the transition probability matrix of the joint Markov process (x_t, z_t). Let

$$\mathbf{Q}(\mathbf{r}_t, \mathbf{y}_t^q) = \mathrm{Diag}[Q_{11}, Q_{12}, Q_{21}, Q_{22}]\,(\mathbf{r}_t, \mathbf{y}_t^q)$$

where

$$\begin{aligned} &Q_{ij}\left(\mathbf{r}_t, \mathbf{y}_t^q\right) \\ &\quad = \int_{\mathcal{Q}^{-1}(\mathbf{r}_t, \mathbf{y}_t^q)} f\left(y_1 - g_1(s_i)s_j^z, y_2 - g_2(s_i)s_j^z\right) dy_1 dy_2 \end{aligned}$$

with $\mathcal{Q}^{-1}(\mathbf{r}_t, \mathbf{y}_t^q) = \{(y_1, y_2) : \mathcal{Q}(\mathbf{r}_t, y_1, y_2) = \mathbf{y}_t^q\}$. The recursion for the information state is

$$\begin{aligned} \theta_{t+1} &= \frac{1}{\varphi_{t+1}} \mathbf{Q}\left(\mathbf{r}_{t+1}, \mathbf{y}_{t+1}^q\right) \mathbf{D}^T \theta_t \\ &\triangleq \frac{1}{\varphi_{t+1}} \mathbf{T}\left(\mathbf{r}_{t+1}, \mathbf{y}_{t+1}^q\right) \theta_t \end{aligned}$$

where $\varphi_{t+1} = \|Q(\mathbf{r}_{t+1}, \mathbf{y}_{t+1}^q)\mathbf{D}^T\theta_t\|_1$. The conditional cost is

$$\begin{aligned} c(\theta_t) = \beta(s_1 - s_2)^2[\theta(1) + \theta(2)][\theta(3) + \theta(4)] \\ + (s_1^z - s_2^z)^2[\theta(1) + \theta(3)][\theta(2) + \theta(4)]. \end{aligned}$$

As in Section III, the Bellman equation may be written in the following form:

$$\begin{aligned} &\lambda + h(\theta) \\ &\quad = \min_{\mathbf{r}\in\mathcal{L}} \left[c(\theta) + \sum_{\mathbf{y}^q} \|\mathbf{T}(\mathbf{r}, \mathbf{y}^q)\theta\|_1 h\left(\frac{\mathbf{T}(\mathbf{r}, \mathbf{y}^q)\theta}{\|\mathbf{T}(\mathbf{r}, \mathbf{y}^q)\theta\|_1}\right) \right] \end{aligned} \tag{4.2}$$

where $\mathcal{L} = (L_{d_1}^b \times L_{d_2}^t) \cup (L_{d_2}^t \times L_{d_1}^b)$. Furthermore, the analysis as well as the numerical scheme in previous sections can be extended to accommodate (4.2) in a straightforward manner. The details will not be repeated here.

V. Simulation Studies

In this section, we present three different simulation experiments to illustrate the performance of our algorithms presented in Sections III and IV.

A. Estimating a Two State Markov Chain Via Two Sensors

In these simulations, the Markov chain x_t has two states $s_1 = 0$ and $s_2 = 2$. The transition matrix for x_t is

$$\mathbf{P} = \begin{bmatrix} 0.8 & 0.2 \\ 0.4 & 0.6 \end{bmatrix}$$

and the two noise components are independent and Gaussian with $\sigma_1^2 = \sigma_2^2 = 0.5$.

For the two state case, we have $\theta = [\theta(1), \theta(2)]^T$ with $\theta(1) + \theta(2) = 1$. The differential cost h is parametrized in terms of $\theta(1)$ and denoted as $h(\theta(1))$. The step size $1/N = 0.01$ is taken for discretization of θ. The set $L_d = \{0, 0.1, 0.2, \ldots, 1.9, 2.0\}$ is used in the (3.10). The pair (λ, h) is computed using the relative value iteration algorithm by 20 iterates. This algorithm essentially is a variation of the standard value iteration method. However, the iterates obtained by the relative value iteration

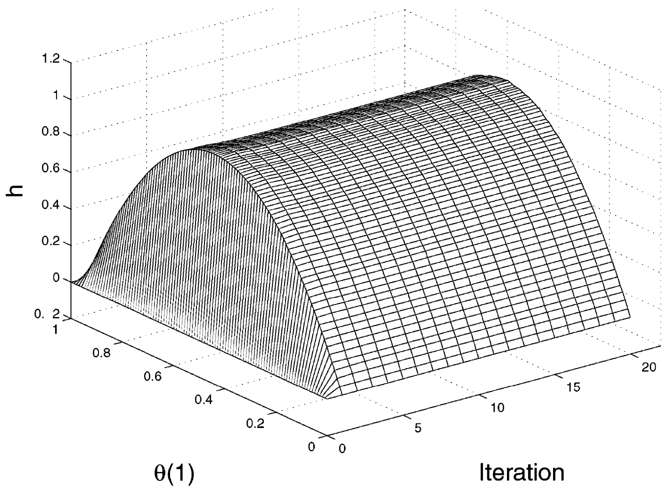

Fig. 2. Iteration for h; each slice corresponds to the curve of h at a fixed iterate.

method are bounded and they converge under a relaxed version of H1), which requires $\mathbf{T}(\mathbf{r}, \mathbf{y}^q)$ to be primitive. (For details on the implementation of this algorithm, see ([4], pp. 320–321).) Fig. 2 shows the convergence of the differential cost. The optimal cost λ converges to 0.11996 as shown in Fig. 3(a).

The cost for static quantizers is computed where the quantizer thresholds for the two sensors are parametrized by a common scalar parameter $r \in L'_d = \{0.5, 0.6, \ldots, 1.5\}$. The associated costs for different r are displayed in Fig. 3(b). For comparison, the solid line indicates the optimal cost 0.11996 for the dynamic quantization.

B. Tracking Multiple-State Slow Markov Chains

1) Single Sensor Estimation: In this example only a single sensor is employed for estimating a slow Markov chain with measurement $y_t = x_t + w_t$. With the quantization parameter $r_t \in \mathbb{R}$, the output is: $y_t^q = a_1$ if $y_t < r_t$, and $y_t^q = a_2$ if $y_t \geq r_t$. Here the i.i.d. Gaussian noise w_t has variance σ^2. x_t has three states $\{s_1 = 0, s_2 = 1, s_3 = 2.5\}$ and transition matrix

$$\mathbf{P} = \begin{bmatrix} 0.9 & 0.1 & 0 \\ 0.1 & 0.8 & 0.1 \\ 0 & 0.15 & 0.85 \end{bmatrix}.$$

We compute the different cases for the binary and ternary quantizers, and in Table I the cost for each scenario refers to the long term average of the mean square estimation error for the associated quantizer.

For the static binary quantizers with r chosen from $\{0.5, 0.6, \ldots, 2.0\}$ (with step size 0.1), the lowest attainable cost is listed in Table I with the associated value for r. It is shown when the noise variance decreases, the relative improvement in performance by dynamic quantization increases.

We also give the cost for a static ternary quantizer which partitions the range space of the observations by a pair of parameters $r_1 = 0.5$ and $r_2 = 1.75$, each of which corresponds to the midpoint of two adjacent states of the Markov chain. It is seen that in the case $\sigma^2 = 0.1$, the cost for the dynamic binary quantizer is nearly twice and four times of that for the static ternary quantizer and the unquantized case, respectively. An intuitive interpretation for the difference between the costs is that when the binary quantizer is employed, the residual uncertainty associated with the state of the Markov chain is higher. For example, when the fusion center receives a symbol for the event $\{y_t \geq 0.5\}$ (assuming $r = 0.5$ is used by the dynamic binary quantizer for that time instant), neither $s_2 = 1$ nor $s_3 = 2.5$ can be asserted with a high likelihood as the true state by using that received symbol alone.

The fourth and fifth columns in Table I compare the performance improvement for the ternary quantizer due to dynamic quantization.

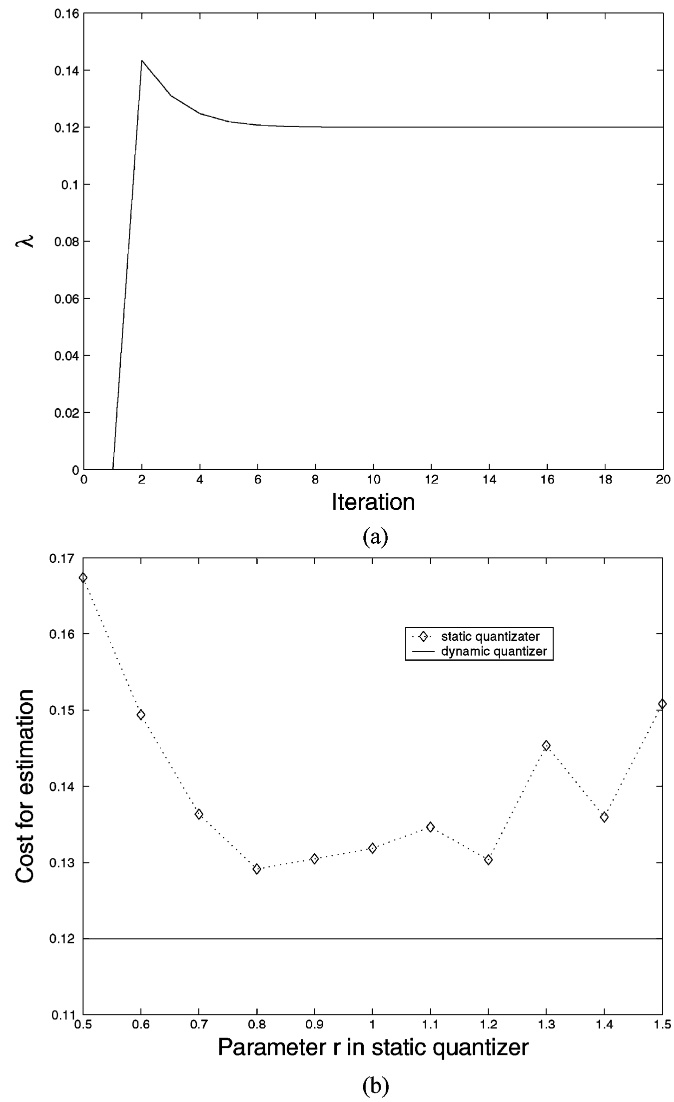

Fig. 3. (a) Convergence of the cost during iteration to 0.11996; (b) the lowest cost attained by static quantization in L'_d is 0.129144 with $r = 0.8$ for two sensors.

TABLE I
COSTS COMPUTED BY 50 ITERATES

σ^2	static binary	dynamic binary	static ternary	dynamic ternary	unquantized
0.5	0.277563 ($r = 1.5$)	0.25842	0.22508	0.20789	0.17964
0.3	0.228534 ($r = 1.5$)	0.19482	0.16026	0.139441	0.114328
0.1	0.171597 ($r = 1.7$)	0.081356	0.041788	0.027429	0.022818

TABLE II
SIMULATION FOR THE MEAN-SQUARE ESTIMATION ERROR

σ^2	mean square error	relative error
0.5	0.178616	0.57%
0.3	0.113857	0.41%
0.1	0.023208	1.71%

The last column gives the mean square estimation error without quantization, which may be numerically computed by taking dense partition for the measurement variable y_t in the associated Bellman equation. In fact, on the subset $[-2, 4.5]$ of the range space of y_t, we take a dense grid of step size 0.1, and use a sparser partition in the rest part. A total number of 95 points is placed in the interval $(-\infty, \infty)$.

In Table II, we compute the long term average of the mean square error in state estimation without quantization by Monte Carlo simulations. The state estimates are obtained by an observation sequence of y_t with 10^5 steps via the standard recursive algorithm for conditional probability estimates. The mean square error is computed by averaging on 30 runs of the simulation. The second column illustrates the relative error $|c_{\rm sim} - c_{\rm iter}|/c_{\rm iter} \times 100\%$ between the simulation based cost (denoted as $c_{\rm sim}$) and the relative value iteration based one (denoted as $c_{\rm iter}$) as given in the last column in Table I which utilizes a dense partition to approximate an analogous observation scheme. It is clearly seen that the results obtained by these two methods are consistent. The additional advantage of the (relative) value iteration approach is that its result does not depend on sample paths.

We briefly summarize the numerical findings for this single sensor tracking problem. Overall, dynamic quantization leads to considerable performance improvement. As the noise variance decreases, the absolute estimation error consistently decreases for both static and dynamic quantization. However, with a lower noise level, dynamic quantization yields an even higher relative improvement, and an intuitive interpretation is that under such a condition the fusion center can better extract information from the sensor messages and hence more effectively guide the quantization threshold setting at the sensor nodes.

In addition, the increase (for instance, from binary to ternary) of the number of quantization levels can efficiently reduce the estimation error. However, this is at the expense of the scarce sensor data rate.

2) Multiple-Sensor Estimation: Now we consider multiple sensors for state estimation. Our purpose is to study the performance improvement either by increasing the number of sensors or by employing dynamic quantization, and compare the difference between these two means. The underlying three state Markov chain x_t is the same as in Section V-B-1. The measurement noise for all sensors are i.i.d. Gaussian with variance $\sigma^2 = 0.3$, regardless of the number of sensors.

First we consider two sensors with static binary quantization. For a heterogeneous quantization scheme using $r_1 = 0.5$ for the first sensor and $r_2 = 1.75$ for the second sensor, the obtained cost is 0.151516. With a homogeneous quantization where the parameters r_1 and r_2 take identical values, the lowest cost is 0.143506 attained at $r_1 = r_2 = 1.4$ which is selected from the set $\{0.6, 0.7, 0.8, \ldots, 2.0\}$. By dynamic binary quantization, the optimal cost is 0.115081 for which the two quantizers are optimized using the set $\{0.6, 0.8, \ldots, 2.0\}$ with a step size 0.2.

As shown by the fifth column in Table I, a single dynamic ternary sensor has a slightly better performance than two static binary sensors, where the former and the latter require to transmit three and four symbols, respectively, in the process of estimation.

For three static homogeneous sensors, the lowest cost 0.09553 is attained at $r_i = 1.3, 1 \le i \le 3$, which is selected from the set $\{0.6, 0.7, 0.8, \ldots, 2.0\}$ with a step size 0.1.

It is seen that the performance of two dynamic binary sensors is comparable to that of three static binary sensors. The latter performs slightly better; however, this is at the cost of a higher total rate (associated with six symbols) for the sensors. This kind of higher rate requirement is usually demanding under low-power conditions for the wireless link from sensors to the fusion center.

Note that in this paper, we do not explicitly address energy budget issues at the sensor nodes. The implementation of a dynamic feedback scheme from the fusion center reduces the number of sensor nodes required, thus reducing energy and bandwidth requirement at the sensor nodes which is crucial. The fusion center is assumed to have an energy supply that is sufficient to allow frequent feedback communication to the sensor nodes.

C. State Estimation for Mode-Dependent Systems

In these simulations we consider the mode-dependent system model given by (4.1), where x_t and z_t have two and three states, respectively. The system is specified as follows: $S = \{1, 2\}, S_z = \{0.8, 2, 3.2\}$, and $g_1(1) = 1, g_2(2) = 1.1, g_1(2) = g_2(1) = 0.25$. The transition matrices for x_t and z_t are given by

$$\mathbf{P} = \begin{bmatrix} 0.85 & 0.15 \\ 0.1 & 0.9 \end{bmatrix}$$
$$\mathbf{P}^z|_{x_t=1} = \begin{bmatrix} 0.8 & 0.1 & 0.1 \\ 0.6 & 0.3 & 0.1 \\ 0.2 & 0.1 & 0.7 \end{bmatrix}$$
$$\mathbf{P}^z|_{x_t=2} = \begin{bmatrix} 0.4 & 0.3 & 0.3 \\ 0.2 & 0.3 & 0.5 \\ 0.0 & 0.2 & 0.8 \end{bmatrix}.$$

The weight $\beta = 0.2$. Let $L^b_{d_1} = \{0.4, 0.6\}$ and $L^t_{d_2} = \{(\gamma_1, \gamma_2) \in D_1 \times D_2\}$, where $D_1 = \{1, 1.5\}$ and $D_2 = \{2, 2.5\}$. The noise sequences for all sensors are i.i.d. Gaussian with covariance $\sigma^2 I_2 = I_2$.

In the rate allocation problem, the quantizer is optimized using $\mathcal{L} = (L^b_{d_1} \times L^t_{d_2}) \cup (L^t_{d_2} \times L^b_{d_1})$. The optimal cost is computed as $\lambda = 0.43198$ using a step size of 0.05 for the information state in 30 iterations.

For comparison, an optimal dynamic quantization without rate allocation is also computed and the quantizer is optimized using $L^b_{d_1} \times L^t_{d_2}$, where the first sensor is always binary. The resulting optimal cost is $\widetilde{\lambda} = 0.49838$ which is clearly suboptimal.

VI. Conclusion

This paper considers dynamic quantization and rate allocation problems in a sensor network with a fusion center where the sensors provide noisy quantized measurements of the state of a source modeled by a finite state Markov chain. A stochastic control approach is used to design the quantizer thresholds and the optimal data rates, which leads to the minimization of the long term average of the mean square estimation error for the underlying Markov chain. The optimization of the network performance is achieved by feedback from the fusion center to the sensor nodes. This is a potentially useful infrastructure for the sensors to adapt better to their operating environment. Within the stochastic control framework, the quantizer and the associated cost may be computed by iterative algorithms associated with the well-known MDP framework. The system performance improvements resulting from dynamic quantization and rate allocation are illustrated through simulation studies.

In this paper, the advantage of feedback is numerically demonstrated in networks with a relatively small number of nodes, and this further suggests the potential of reducing the sensor number by dynamic quantization. For future research, it is of interest to study, analytically or by use of simulations, for achieving a prescribed performance in a large sensor network monitoring certain dynamic activities, to what extent the sensor number may be reduced by fusion center feedback.

Appendix

A. Proof of Proposition 3.2

Under H1), for any sequence $\{\mathbf{r}_t, t \geq 1\}$ taking values in L_d^M, the exponential stability of the (3.1) holds with regard to initial conditions, i.e., there exist $C > 0$ and $\alpha \in (0,1)$, both independent of $\{\mathbf{r}_t, t \geq 1\}$, such that

$$|\theta_t - \bar{\theta}_t| \leq C\alpha^t |\theta_1 - \bar{\theta}_1| \tag{A1}$$

where θ_t and $\bar{\theta}_t$ are two solutions to (3.1) with different initial conditions θ_1 and $\bar{\theta}_1$, respectively (see, e.g., [2]).

Since $c(\theta)$ is continuously differentiable with respect to θ, its first order derivative $dc(\theta)/d\theta$ is bounded on the compact set S_1 which implies $c(\theta)$ is Lipschitz continuous on S_1, i.e., there exists $K > 0$ such that for all $\theta_1, \theta_2 \in S_1$

$$|c(\theta_1) - c(\theta_2)| \leq K|\theta_1 - \theta_2|. \tag{A2}$$

We define the infinite horizon discounted optimal cost $v_d^\rho(\theta) = \inf_{(\mathbf{r}_t, t\geq 1)} E[\sum_{t=1}^{\infty} \rho^t c(\theta_t)|\theta_1 = \theta]$ where $\rho \in (0,1)$. By use of (A1) and (A2), for any $\theta, \bar{\theta} \in S_1$ we apply a comparison technique as in [10] to obtain

$$\begin{aligned}
&|v_d^\rho(\theta) - v_d^\rho(\bar{\theta})| \\
&\quad = \left| \inf_{\{\mathbf{r}_t, t\geq 1\}} E\left[\sum_{t=1}^{\infty} \rho^t c(\theta_t)|\theta_1 = \theta\right] \right. \\
&\qquad \left. - \inf_{\{\mathbf{r}_t, t\geq 1\}} E\left[\sum_{t=1}^{\infty} \rho^t c(\bar{\theta}_t)|\bar{\theta}_1 = \bar{\theta}\right] \right| \\
&\quad \leq \sup_{\{\mathbf{r}_t, t\geq 1\}} E\left[\sum_{t=1}^{\infty} \rho^t |c(\theta_t) - c(\bar{\theta}_t)||\theta_1 = \theta, \bar{\theta}_1 = \bar{\theta}\right] \\
&\quad \leq \sup_{\{\mathbf{r}_t, t\geq 1\}} E\left[\sum_{t=1}^{\infty} \rho^t K|\theta_t - \bar{\theta}_t||\theta_1 = \theta, \bar{\theta}_1 = \bar{\theta}\right] \\
&\quad \leq \sup_{\{\mathbf{r}_t, t\geq 1\}} E\left[\sum_{t=1}^{\infty} K\rho^t C\alpha^t|\theta_1 - \bar{\theta}_1||\theta_1 = \theta, \bar{\theta}_1 = \bar{\theta}\right] \\
&\quad \leq \frac{\alpha CK|\theta - \bar{\theta}|}{1-\alpha}
\end{aligned} \tag{A3}$$

which implies equicontinuity (in $\theta \in S_1$) of $v_d^\rho(\theta)$ w.r.t. all $\rho \in (0,1)$, and also the uniform boundedness of the difference term $|v_d^\rho(\theta) - v_d^\rho(\bar{\theta})|$ with regard to all $\theta, \bar{\theta} \in S_1$ and $\rho \in (0,1)$. Hence, it follows from the standard results in [9] that (3.9) has a bounded solution $h(\theta)$ and the associated $\lambda \in \mathbb{R}$. □

B. Proof of Proposition 3.3

Recall that $\mathbf{T}(\mathbf{r}_t, \mathbf{y}_t^q) = \mathbf{Q}(s_1, \ldots, s_n, \mathbf{r}_t, \mathbf{y}_t^q)\mathbf{P}^T$. Note also that $\|\zeta\|_1 + \|\hat{\zeta}\|_1 = \|\zeta + \hat{\zeta}\|_1$, for any two entry-wise nonnegative vectors ζ and $\hat{\zeta}$. Hence, for $\theta \in S_1$

$$\begin{aligned}
&\sum_{\mathbf{y}^q} \|\mathbf{T}(\mathbf{r}, \mathbf{y}^q)\theta\|_1 \\
&\quad = \sum_{\mathbf{y}^q} \|\mathbf{Q}(s_1, \ldots, s_n, \mathbf{r}, \mathbf{y}^q)\mathbf{P}^T\theta\|_1 \\
&\quad = \left\| \sum_{\mathbf{y}^q} \mathbf{Q}(s_1, \ldots, s_n, \mathbf{r}, \mathbf{y}^q)\mathbf{P}^T\theta \right\|_1
\end{aligned} \tag{A4}$$

where all involved vectors and matrices have nonnegative entries.

On the other hand, by (3.2) and (3.3) it follows that

$$\sum_{\mathbf{y}^q} \mathbf{Q}(s_1, \ldots, s_n, \mathbf{r}, \mathbf{y}^q) = I_n,$$

which combined with (A4) gives

$$\sum_{\mathbf{y}^q} \|\mathbf{T}(\mathbf{r}, \mathbf{y}^q)\theta\|_1 = \|\mathbf{P}^T\theta\|_1 = 1 \tag{A5}$$

since $\theta \in S_1$. This completes the proof. □

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